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# CALCULATION, BY NONLINEAR MECHANICS, OF THE ENERGY DISSIPATED IN AN OSCILLATING BODY

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[A digest]

In the theory of oscillations, problems on oscillatory damping are very important and practical. One of the basic factors influencing oscillatory damping in a viscous real oscillating system is the dissipation of energy in the material of the oscillating body.

In stress-strain diagrams the process of energy dissipation, for a single oscillatory cycle, in a body possessing the property of energy dissipation is expressed in the form of a certain closed loop bound by two smooth curves, namely the so-called hysteresis loop, or folium.

The area of this hysteresis loop expresses, in a certain suitable scale, the amount of heat energy dissipated per unit volume of material per oscillatory cycle. This energy, expressible as a function of normal or tangent stresses, is called the "dissipation energy in a body executing oscillations," or "the internal damping of oscillations," and possesses the dimensions of: kilogram - centimeter per cubic centimeter. Energy dissipation per unit volume of the body executing oscillations must be calculated from those properties of the body that do not depend upon the form and dimensions of the oscillating body.

The problem of energy dissipation in an oscillating body has been considered by many researchers in physics and mechanics. The basic works devoted to the problem of damping were carefully analyzed in 1938 by N. N. Davidenkov (see "On the Dissipation of Energy in Matter," ZhTF, Vol VIII, No 6).

Davidenkov pointed out certain ideas that permit one to solve this problem of energy dissipation in matter, by considering the true physical notion of energy dissipation in matter.



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Moreover, this work pointed out the falseness of the hypothesis of "viscous friction," which many scientists employed in their investigations into energy dissipation in matter, mainly because this hypothesis simplifies the mathematics of energy dissipation in oscillating bodies. This work also proposed a physical hypothesis, namely, the dependence of energy dissipation in matter upon the magnitude of the stress amplitude, which many experiments have proved correct.

In his work Davidenkov assumed that the equations of oscillations for a viscous system which take into account the dissipation of energy in matter depend upon energy dissipation and are not limited to just one equation. He introduced, therefore, into his consideration two equations, one for the ascending motion and another for the descending motion. That is, according to his assumption, the process of oscillation must be described by the contours of the hysteresis loop.

Such a procedure, however, causes considerable mathematical difficulties, since it is necessary to deal with the solution of nonlinear differential equations whose characteristics are represented by certain closed curvilinear contours.

Davidenkov pointed out these difficulties in his work, in which he suggested that the equations be set up as for an oscillating system with one degree of freedom. In the development of Davidenkov's idea, a new theory for calculating oscillatory systems which takes into account the dissipation of energy in matter was worked out.

This new theory makes the following physical assumptions: In all materials with which one must deal in "vibrotechnics," the actual stress-strain relation deviates slightly from Hook's linear law. That is, there is a slight deviation of the ascending and descending branches of the hysteresis loop from rectilinearity. This circumstance suggests that, in order to solve this weakly nonlinear problem one should employ an approximation method that would permit one, by proceeding from present-day physical notions on the essential nature of energy dissipation in matter, to obtain for a minimum number of approximations a high degree of accuracy in the solution of the problem. Thus, the only suitable method to be used in this case is nonlinear mechanics, which is based on series expansions in small parameters, as developed by Academician N. M. Krylov and Corresponding Member N. N. Bogolyubov for solving various other weakly nonlinear problems in physics and technology (see Introduction to Nonlinear Mechanics, Academy of Sciences Ukrainian SSR, 1937).

As investigations showed, in the solution of various problems of pseudoharmonic oscillations, use of the method of small parameters gives even in the first approximation high accuracy, whereas the second approximation gives the value of the frequency with an accuracy of the order of one percent.

The physical basis for a satisfactory solution in the first approximation of an hysteresis-type problem, for all practical purposes, consists of the following: the first approximation involves the actual area of the hysteresis loop (the area determines energy dissipation) and the second approximation involves the actual shape, or form, itself of the loop (the form influences damping only negligibly).

The fact that nonlinear mechanics solves with a satisfactory degree of accuracy, even in the first approximation, the complicated problem of energy dissipation in matter, speaks well for the effectiveness of the proposed method and its practical worth.



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After some general preliminary remarks, the problem on the dissipation of energy in matter will be solved for a concrete example of a real oscillatory system. With this in mind, consider the forced transverse oscillations, or vibrations, of a cantilever-type beam, with a tensile force on the free end and under the action of an exciting force in the form of a transverse load, uniformly distributed along the length of the beam and varying harmonically.

The following symbols are introduced:

$x$ Coordinate axis, directed along the axis of the beam

$u$ Deflection of the beam from its position of equilibrium

$l$ Beam's length

$b$ Beam's width

$h$ Beam's height

$I$ Moment of inertia of the transverse cross section of the beam

$E$ Normal modulus of elasticity of the material

$m$ Mass per unit length of the beam

$\omega$ Frequency of forced oscillation of the beam

$\omega_c$ Natural frequency of oscillation of the beam

$q_0$ Amplitude of the uniformly-distributed exciting force

$t$ Time

$\varepsilon$ Any small parameter

$\phi$ Phase angle between stress and strain

$N$ Tensile force acting in the direction of the x axis

$Q$ "Intersecting" force

$M$ Bending moment

$\sigma$ Stress

$\xi$ Relative elongation (strain)

The differential equations describing the oscillation of the beam under discussion will be set up on the assumptions that the oscillations occur in one of the beam's main planes of curvature and that the beam's transverse cross section is small in comparison with beam length. Moreover, it is assumed that certain separate nonlinear stress-strain relations exist for the two cases of "load" and "unload" in the material:

$$\left.\begin{aligned}\vec{\sigma} &= E\left|\xi+\vec{f}(\zeta)\right| \\ \overleftarrow{\sigma} &= E\left|\xi+\overleftarrow{f}(\zeta)\right|\end{aligned}\right\}, \qquad (1)$$



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where $\vec{E}f(\zeta)$ and $\overleftarrow{E}f(\zeta)$ are certain increments in stress which characterize the deviation of the ascending and descending branches of the hysteresis loop from linearity, which linear part is expressed by the first term of formula (1) for the determination of stress $\sigma$; that is, $E\xi$ is the linear term.

In accordance with formula (1), the bending moment acting in a cross section of the beam at any moment of time can be expressed as a function of the second derivative of deflection u:

$$-M = M\left(\frac{d^2u}{dx^2}\right) = EI\frac{d^2u}{dx^2} + \varepsilon\vec{\Phi}\left(\frac{d^2u}{dx^2}\right), \tag{2}$$

where $\varepsilon\Phi\left(\frac{d^2u}{dx^2}\right)$ is a certain function characterizing the imperfect elasticity of the material. The value of this function will differ for the ascending and descending movements, depending upon which of the various branches of the hysteresis loop the phase point is traveling on.

Consider the sum of the bending moments acting upon an individual element of the beam. We find:

$$\frac{dM}{dx} = Q - N\frac{du}{dx} \tag{3}$$

On the other hand, we know that:

$$\frac{dQ}{dx} = \frac{d^2M}{dx^2} = -q \tag{4}$$

Then in view of formula (2), we can write:

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{d^2u}{dx^2}\right] + \frac{\partial^2}{\partial x^2}\left[\varepsilon\Phi\left(\frac{\partial^2u}{\partial x^2}\right)\right] - N\frac{\partial^2u}{\partial x^2} = q_1 \tag{5}$$

This differential equation of a beam subjected to a uniformly distributed load $q_1$, is used to obtain the equation of transverse oscillation of the beam.

We employ D'Alembert's principle, according to which the sum of all forces acting on an element of the beam, both external and elastic, and also the forces of inertia, must be set equal to zero.

With this in mind, the force of inertia and the external exciting force will be:

$$q_i = -m\frac{\partial^2u}{\partial t^2} \tag{5'}$$

and $$q_t = \varepsilon q\cos\omega t$$

Making use of D'Alembert's principle and disregarding the "intersecting" force and the inertia of rotation of the mass, we obtain the following differential equation of the transverse (lateral) oscillations of the beam under discussion:

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2u}{\partial x^2}\right] + \frac{\partial^2}{\partial x^2}\left[\varepsilon\vec{\Phi}\left(\frac{\partial^2u}{\partial x^2}\right)\right] - N\frac{\partial^2u}{\partial x^2} + m\frac{\partial^2u}{\partial t^2} - \varepsilon q\cos\omega t = 0. \tag{6}$$



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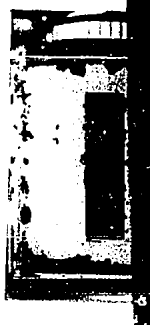

Let us now consider the forced oscillations that are set up and the energy that is dissipated because of losses in the oscillating body (the value of the energy lost is assumed to be rather small). Let us assume that the external periodic force necessary to maintain oscillations is also rather small, which fact is indicated by the presence of the small parameter epsilon in the form of a factor, or coefficient, both in the term designating the energy loss in the material and also in the term expressing the amplitude of the exciting force; that is, in the second and last terms of equation (6).

Moreover, it is necessary to keep in mind that the values of the functional $\varepsilon \frac{\partial^2}{\partial x^2}\left[\Phi\left(\frac{\partial^2 u}{\partial x^2}\right)\right]$, which allows for the dissipation of energy in the material for increases in the load, and for its decreases, will be different, which fact is indicated by the arrows above this term and directed in different ways.

Let us introduce the relative, that is, reduced or dimensionless-coordinate $\zeta = x/L$ and the relative deflection $u^* = u/L$; then equation (6) can be rewritten in terms of relative quantities and becomes as follows:

$$\frac{\partial^4 u^*}{\partial \zeta^4} + \frac{\varepsilon}{EI}\frac{\partial^2}{\partial \zeta^2}\left[\Phi\left(\frac{\partial^2 u^*}{\partial \zeta^2}\right)\right] - \frac{Nl^2}{EI}\frac{\partial^2 u^*}{\partial \zeta^2} + \frac{m l^4}{EI}\frac{\partial^2 u^*}{\partial t^2} - \frac{\varepsilon q l^3}{EI}\cos \omega t = 0. \qquad (7)$$

In order to solve this equation, we employ the approximate method afforded by nonlinear mechanics, which involves series expansions in small parameters. In following this method we shall assume that the reduced deflection $u^*(\zeta, t)$, the frequency $\omega$ of oscillation, and the phase displacement $\phi$ can be represented as series expansions in $\varepsilon$, thus:

$$u^*(\zeta, t) = \varphi(\zeta)\, a \cos(\omega t + \phi) + \varepsilon u_1(\zeta, t) + \varepsilon^2 u_2(\zeta t) + \ldots . \qquad (8)$$

$$\omega^2 = \omega_0^2 + \varepsilon \Delta_1 + \varepsilon^2 \Delta_2 + \ldots \qquad (9)$$

$$\phi = \phi_0 + \varepsilon \phi_1 + \varepsilon^2 \psi_2 + \ldots \qquad (10)$$

where $a = q/L$ is the relative amplitude of deflection at the end of the cantilever-type beam

NOTE: The remainder of the work is concerned with the mathematical operations of inserting the above series into equation (7) and resolving. Then the various factors of the terms in these series are determined according to the usual small-parameter method of Poincare and others.

At the end, a concrete physical example is taken, where the above letters, L, b, h, E, n, N, q, etc., are assigned certain actual values. The frequency ratio $\omega/\omega_0$ is then determined.

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